

Detailed Balance and Sea-Quark Flavor Asymmetry of Proton

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Abstract

In this study, the proton is taken as an ensemble of quark-gluon Fock states. Using the principle of detailed balance, the probabilities of finding every Fock states of the proton are obtained without any parameter. A new origin of the light flavor sea quark asymmetry, i.e., $\bar{u} \neq \bar{d}$, is given as a pure statistical effect. It is found that $\bar{d} - \bar{u} \approx 0.124$, which is in surprisingly agreement with the experimental observation.

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The composition of hadrons in terms of the fundamental quark and gluon degrees of freedom is a central focus of hadronic physics. In the light-cone Fock state description of bound states [1], each physical hadron state is expanded by a complete set of quark-gluon Fock states as

$$|h\rangle = \sum_{i,j,k} c_{i,j,k} |\{q\}, \{i, j, k\}, \{l\}\rangle, \quad (1)$$

where $\{q\}$ represents the valence quarks of the hadron, i is the number of quark-antiquark $u\bar{u}$ pairs, j is the number of quark-antiquark $d\bar{d}$ pairs, k is the number of gluons, and $\{l\}$ represents the heavy flavor (s , c , b , and t) $q\bar{q}$ pairs which will be neglected at first in this study. There have been many progress in understanding the hadron structure in terms of the underlying quark and gluon degrees of freedom. For example, we can obtain the lowest valence Fock state $|\{q\}\rangle$ by the quark model [2]. We can also model the next higher $|\{q\}, \{q\bar{q}\}\rangle$ Fock states by the energetically-favored baryon-meson fluctuations [3]. The Fock states with gluons, $|\{q\}, \{g\}\rangle$, have been also studied [4]. However, we are still lacking a comprehensive understanding concerning the completed set of Fock states for hadrons, even for the proton.

In this work, we will provide a complete set of Fock states for the proton from a simple statistical consideration. By (1), it means that we take the proton as an ensemble of quark-gluon Fock states. The probability to find the proton in the $|\{q\}, \{i, j, k\}\rangle$ Fock state is

$$\rho_{i,j,k} = |c_{i,j,k}|^2, \quad (2)$$

where $\rho_{i,j,k}$ satisfies the normalization condition,

$$\sum_{i,j,k} \rho_{i,j,k} = 1. \quad (3)$$

Therefore the parton numbers of quarks and gluons in the proton are

$$\begin{aligned}
u &= u_v + \sum_{i,j,k} i \rho_{i,j,k}, \\
d &= d_v + \sum_{i,j,k} j \rho_{i,j,k}, \\
\bar{u} &= \sum_{i,j,k} i \rho_{i,j,k}, \\
\bar{d} &= \sum_{i,j,k} j \rho_{i,j,k}, \\
g &= \sum_{i,j,k} k \rho_{i,j,k},
\end{aligned} \tag{4}$$

where $u_v = 2$ and $d_v = 1$ are the valence quark numbers of the proton. These parton numbers can be measured by deep inelastic scattering of leptons on the proton target. The quarks and gluons in the Fock states are the “intrinsic” partons of the proton, since they are multi-connected non-perturbatively to the valence quarks [3]. Such partons are different from the “extrinsic” partons generated from the QCD hard bremsstrahlung and gluon-splitting as part of the lepton scattering interaction.

Using statistical property of the ensemble, the probability $\rho_{i,j,k}$ can be calculated without any parameter in the following way. Let us trace one system (a proton) in the big ensemble (a set of quark-gluon Fock states). At the time $t = 0$, the system is in one state of a subensemble ($A = |\{q\}, \{i, j, k\}\rangle$). After a while, at the time $t = \delta t$, the state of this system has three probabilities: (1) does not change; (2) changes to another state that belongs to the same subensemble (A); (3) changes to a state that belongs to a different subensemble ($B = |\{q\}, \{i', j', k'\}\rangle$). In the last case, the density of subensemble (A) decreases ($\rho_A \downarrow$). On the other hand, some states in subensemble (B) will be changed to subensemble (A) during the same period, and increases the density of subensemble (A) ($\rho_A \uparrow$). These two processes, which are in opposite direction for each other, just compensate for each other, so ρ_A keeps unchanged. This is the so called detailed balance, which demands every two subensembles to balance with each other. So we have

$$\rho_{i,j,k} |\{q\}, \{i, j, k\}\rangle \xrightleftharpoons{\text{balance}} \rho_{i',j',k'} |\{q\}, \{i', j', k'\}\rangle, \tag{5}$$

$$\rho_{i,j,k} N \{|\{q\}, \{i, j, k\}\rangle \rightarrow |\{q\}, \{i', j', k'\}\rangle\} \equiv \rho_{i',j',k'} N \{|\{q\}, \{i', j', k'\}\rangle \rightarrow |\{q\}, \{i, j, k\}\rangle\}, \tag{6}$$

$$\frac{\rho_{i,j,k}}{\rho_{i',j',k'}} \equiv \frac{N \{ |\{q\}, \{i', j', k'\}\rangle \rightarrow |\{q\}, \{i, j, k\}\rangle \}}{N \{ |\{q\}, \{i, j, k\}\rangle \rightarrow |\{q\}, \{i', j', k'\}\rangle \}}, \quad (7)$$

where $N\{A \rightarrow B\}$ implies the number of processes that transfer A into B , i.e., the transfer rate of $A \rightarrow B$.

In order to know $\rho_{i,j,k}$, the probability of finding the proton in the Fock state A , all what we need to know is the ratio between the transfer rate of $A \rightarrow B$ and the transfer rate of $B \rightarrow A$. In the realistic situation, the transfer rate is very complicated and it involves many factors, such as: the proton size, the quark mass, the parton numbers, the quantum numbers (color, spin, flavor), cross sections of sub-processes, exchange symmetry (Pauli blocking), the velocity and even the details of the quantum wave functions of partons. Here, among all these factors, we only take into account the number of partons from a pure statistical consideration. We also neglect the interaction involving $g \Leftrightarrow gg$ at first. We will find, surprisingly, that our simplified calculation gives the correct light flavor sea-quark asymmetry, i.e., $\bar{u} \neq \bar{d}$, which has been observed in experiments [5–10]. This implies that the number of valence quarks controls the statistical property of the proton.

The transfer between two subensembles has two ways: splitting and recombination. Splitting rate is proportional to the number of partons that may split. Recombination rate is proportional to both the number of those two kinds of partons that may recombine. So we have the following two kinds of relations:

(1) Detailed balance involving $q \Leftrightarrow qg$ gives the relation

$$\begin{array}{c} 3 \\ |ud\rangle \rightleftharpoons |udg\rangle, \end{array} \quad (8)$$

$$1 \times 3$$

$$\begin{array}{c} 3 \\ |udg\rangle \rightleftharpoons |udgg\rangle, \end{array} \quad (9)$$

$$2 \times 3$$

$$\begin{array}{c} 3 \\ |udgg\rangle \rightleftharpoons |udggg\rangle, \end{array} \quad (10)$$

$$\begin{array}{c} 3 \times 3 \\ \dots \end{array} \quad (11)$$

$$\begin{array}{c} 5 \\ |ud\bar{u}u\rangle \rightleftharpoons |ud\bar{u}ug\rangle, \end{array} \quad (12)$$

$$\begin{array}{c} 1 \times 5 \\ \dots \end{array} \quad (13)$$

$$\begin{array}{c} 3 + 2i + 2j \\ |\{q\}, \{i, j, k-1\}\rangle \rightleftharpoons |\{q\}, \{i, j, k\}\rangle. \end{array} \quad (14)$$

$$(3 + 2i + 2j)k$$

then a general formula can be derived using formula (7),

$$\frac{\rho_{i,j,k}}{\rho_{i,j,k-1}} = \frac{1}{k}, \quad (15)$$

$$\frac{\rho_{i,j,k}}{\rho_{i,j,0}} = \frac{1}{k!}. \quad (16)$$

(2) Detailed balance involving $g \Leftrightarrow \bar{q}q$ gives the relation

$$\begin{array}{c} 1 \\ |udg\rangle \rightleftharpoons |ud\bar{u}u\rangle, \end{array} \quad (17)$$

$$1 \times 3$$

$$\begin{array}{c} 1 \\ |udg\rangle \rightleftharpoons |udd\bar{d}\rangle, \end{array} \quad (18)$$

$$1 \times 2$$

$$\begin{array}{c} 1 \\ |ud\bar{u}ug\rangle \rightleftharpoons |ud\bar{u}u\bar{u}u\rangle, \end{array} \quad (19)$$

$$2 \times 4$$

$$\begin{array}{c} 1 \\ |uud\bar{u}ug\rangle \rightleftharpoons |uud\bar{u}d\bar{d}\rangle, \end{array} \quad (20)$$

$$\begin{array}{c} 1 \times 2 \\ \dots \end{array} \quad (21)$$

$$\begin{array}{c} 1 \\ |\{q\}, \{i-1, j, 1\}\rangle \rightleftharpoons |\{q\}, \{i, j, 0\}\rangle, \end{array} \quad (22)$$

$$\begin{array}{c} i(i+2) \\ 1 \\ |\{q\}, \{i, j-1, 1\}\rangle \rightleftharpoons |\{q\}, \{i, j, 0\}\rangle. \\ j(j+1) \end{array} \quad (23)$$

Then a general formula can be derived using formula (7),

$$\frac{\rho_{i,j,0}}{\rho_{i-1,j,1}} = \frac{1}{i(i+2)}, \quad (24)$$

$$\frac{\rho_{i,j,0}}{\rho_{i,j-1,1}} = \frac{1}{j(j+1)}. \quad (25)$$

Using relation $\rho_{i,j,1} = \rho_{i,j,0}$ gotten from formula (16) to these two new formulae, we get

$$\frac{\rho_{i,j,0}}{\rho_{0,j,0}} = \frac{2}{i!(i+2)!}, \quad (26)$$

$$\frac{\rho_{i,j,0}}{\rho_{i,0,0}} = \frac{1}{j!(j+1)!}, \quad (27)$$

$$\frac{\rho_{i,j,0}}{\rho_{0,0,0}} = \frac{2}{i!(i+2)!j!(j+1)!}. \quad (28)$$

Combing formulae (16) and (28), we obtain the general formula

$$\frac{\rho_{i,j,k}}{\rho_{0,0,0}} = \frac{2}{i!(i+2)!j!(j+1)!k!}. \quad (29)$$

Then from the normalization condition (3) and formula (29), all $\rho_{i,j,k}$ can be calculated as shown in TABLE I.

From TABLE I, we get the numbers of intrinsic gluons and sea quarks of the proton,

$$\bar{u} = \sum_{i,j,k} i\rho_{i,j,k} = 0.308, \quad (30)$$

$$\bar{d} = \sum_{i,j,k} j \rho_{i,j,k} = 0.432, \quad (31)$$

$$g = \sum_{i,j,k} k \rho_{i,j,k} = 0.997, \quad (32)$$

$$\bar{d} - \bar{u} = 0.124. \quad (33)$$

These parton numbers should be considered to work at a scale for the “intrinsic” partons, and the details need further studies. It is very interesting that only from the pure statistical consideration, we get an asymmetry between the light flavor u and d sea quarks, i.e., $\bar{u} \neq \bar{d}$, which has been observed in deep inelastic scattering and Drell-Yan experiments [5–10]. Moreover, the flavor sea-quark asymmetry $\bar{d} - \bar{u}$ can be checked by experiments directly because its Q^2 dependence is small. It is a surprise that our result is in excellent agreement with the recent experimental result [10] $\bar{d} - \bar{u} = 0.118 \pm 0.012$. This good agreement indicates that the principle of detailed balance plays an essential role in the structure of proton.

We now check whether the inclusion of the process $g \Leftrightarrow gg$ can change the above conclusion. We make the similar analysis on the two kinds of relations:

(1) Detailed balance involving $q \Leftrightarrow qg$ and $g \Leftrightarrow gg$ gives the relation

$$\begin{array}{c} 3 \\ |uud\rangle \rightleftharpoons |uudg\rangle, \end{array} \quad (34)$$

$$1 \times 3$$

$$\begin{array}{c} 3+1 \\ |uudg\rangle \rightleftharpoons |uudgg\rangle, \end{array} \quad (35)$$

$$2 \times 3 + 1$$

$$\begin{array}{c} 3+2 \\ |uudgg\rangle \rightleftharpoons |uudggg\rangle, \end{array} \quad (36)$$

$$3 \times 3 + 3$$

$$\dots \quad (37)$$

5

$$|uud\bar{u}u\rangle \rightleftharpoons |uud\bar{u}ug\rangle, \quad (38)$$

 1×5

$$\dots \quad (39)$$

 $3 + 2i + 2j + k - 1$

$$|\{q\}, \{i, j, k-1\}\rangle \rightleftharpoons |\{q\}, \{i, j, k\}\rangle, \quad (40)$$

 $(3 + 2i + 2j)k + C_k^2$

where $C_k^2 = \frac{k(k-1)}{2}$. Then a general formula can be derived using formula (7),

$$\frac{\rho_{i,j,k}}{\rho_{i,j,k-1}} = \frac{3 + 2i + 2j + k - 1}{(3 + 2i + 2j)k + \frac{k(k-1)}{2}}. \quad (41)$$

(2) Detailed balance involving $g \Leftrightarrow \bar{q}q$ has no effect by the process $g \Leftrightarrow gg$ so the Eq.(28) remains unchanged.

Combining (41), (28), and the normalization condition (3), all $\rho_{i,j,k}$ can be calculated as shown in TABLE II. From TABLE II, we get the numbers of intrinsic gluons and sea quarks of the proton,

$$\bar{u} = \sum_{i,j,k} i \rho_{i,j,k} = 0.304, \quad (42)$$

$$\bar{d} = \sum_{i,j,k} j \rho_{i,j,k} = 0.426, \quad (43)$$

$$g = \sum_{i,j,k} k \rho_{i,j,k} = 1.109, \quad (44)$$

$$\bar{d} - \bar{u} = 0.123, \quad (45)$$

which are very close to the situation without considering $g \Leftrightarrow gg$, except for the gluon number with a bigger difference of 10%. Here the flavor sea-quark asymmetry $\bar{d} - \bar{u}$ is also in good agreement with the recent experimental result $\bar{d} - \bar{u} = 0.118 \pm 0.012$.

We point out here that the heavy quark-antiquark pairs are not considered in this paper. Taking them into account is not so simple and may introduce some parameters, and this is beyond the simple spirit of this paper. So we neglect them in this study, though there have been suggestions of “intrinsic” strange [3] and charm [11,12] of the proton. We also need

to check the applicability for the statistical method to model the structure of the nucleon. The total number of intrinsic partons inside the proton is around

$$N = u_{\text{val}} + d_{\text{val}} + u_{\text{sea}} + \bar{u} + d_{\text{sea}} + \bar{d} + g \quad (46)$$

$$= 2 + 1 + 2\bar{u} + 2\bar{d} + g = 5.5, \quad (47)$$

which is a small number of particles for the feasibility of the statistical method. The success of this study suggests us to apply the statistical method to the hadronic structure studies, though its range of applicability should be very limited. It is necessary to apply the statistical method from more sophisticated consideration. Similar application can be also extended to other baryons.

In summary, we studied the proton structure as an ensemble of quark-gluon Fock states. Using the principle of detailed balance, the probabilities of finding the proton in every Fock states are obtained without any parameter. A new origin of the light flavor sea quark asymmetry, i.e., $\bar{u} \neq \bar{d}$, is given as a pure statistical effect. It is found that $\bar{d} - \bar{u} \approx 0.124$, which is in surprisingly agreement with the experimental observation.

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TABLES

TABLE I. The probabilities, $\rho_{i,j,k}$, of finding the quark-gluon Fock states of the proton, calculated using the principle of detailed balance without any parameter. $|\{q\}, \{i, j, k\}\rangle$ is the subsemble of Fock states, i is the number of $u\bar{u}$ quark pairs, j is the number of $d\bar{d}$ pairs, and k is the number of gluons.

i	j	$ \{q\}, \{i, j, 0\}\rangle$	$\rho_{i,j,0}$	$\rho_{i,j,1}$	$\rho_{i,j,2}$	$\rho_{i,j,3}$	$\rho_{i,j,4}$...
0	0	$ uud\rangle$	0.167849	0.167849	0.083924	0.027975	0.006994	...
1	0	$ uud\bar{u}u\rangle$	0.055950	0.055950	0.027975	0.009325	0.002331	...
0	1	$ uudd\bar{d}\rangle$	0.083924	0.083924	0.041962	0.013987	0.003497	...
1	1	$ uud\bar{u}udd\bar{d}\rangle$	0.027975	0.027975	0.013987	0.004662	0.001166	...
0	2	$ uudd\bar{d}\bar{d}\bar{d}\rangle$	0.013987	0.013987	0.006994	0.002331	0.000583	...
2	0	$ uud\bar{u}u\bar{u}u\rangle$	0.006994	0.006994	0.003497	0.001166	0.000291	...
1	2	$ uud\bar{u}udd\bar{d}\bar{d}\rangle$	0.004662	0.004662	0.002331	0.000777	0.000194	...
2	1	$ uud\bar{u}u\bar{u}udd\bar{d}\rangle$	0.003497	0.003497	0.001748	0.000583	0.000146	...
0	3	$ uudd\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}\rangle$	0.001166	0.001166	0.000583	0.000194	0.000049	...
3	0	$ uud\bar{u}u\bar{u}u\bar{u}u\rangle$	0.000466	0.000466	0.000233	0.000078	0.000019	...
...

TABLE II. The probabilities of finding the quark-gluon Fock states of the proton, with the process $g \Leftrightarrow gg$ considered.

i	j	$ \{q\}, \{i, j, 0\}\rangle$	$\rho_{i,j,0}$	$\rho_{i,j,1}$	$\rho_{i,j,2}$	$\rho_{i,j,3}$	$\rho_{i,j,4}$...
0	0	$ uud\rangle$	0.158885	0.158885	0.090791	0.037830	0.012610	...
1	0	$ uud\bar{u}\bar{u}\rangle$	0.052962	0.052962	0.028888	0.011234	0.003457	...
0	1	$ uudd\bar{d}\rangle$	0.079442	0.079442	0.043332	0.016851	0.005185	...
1	1	$ uud\bar{u}udd\rangle$	0.026481	0.026481	0.014123	0.005296	0.001558	...
0	2	$ uudddd\bar{d}\rangle$	0.013240	0.013240	0.007062	0.002648	0.000779	...
2	0	$ uud\bar{u}u\bar{u}\bar{u}\rangle$	0.006620	0.006620	0.003531	0.001324	0.000389	...
1	2	$ uud\bar{u}udd\bar{d}\bar{d}\rangle$	0.004413	0.004413	0.002323	0.000852	0.000243	...
2	1	$ uud\bar{u}u\bar{u}udd\rangle$	0.003310	0.003310	0.001742	0.000639	0.000183	...
0	3	$ uudddd\bar{d}\bar{d}\bar{d}\bar{d}\rangle$	0.001103	0.001103	0.000581	0.000213	0.000061	...
3	0	$ uud\bar{u}u\bar{u}u\bar{u}\bar{u}\rangle$	0.000441	0.000441	0.000232	0.000085	0.000024	...
...